Clarification of Gravitational Potential Notation

Clarification of Gravitational Potential Notation used in gravitation acceleration equation

In the formulation of Pawan Upadhyay's Pressure Curvature Law of Gravity (PPC), gravitational acceleration is written as:

$g = -\nabla\varphi$

Here, the lowercase phi (φ) denotes the gravitational potential used in the PPC formulation.

In the weak-field limit of Einstein's field equations, including the weak-field treatment of a perfect fluid, the gravitational potential is conventionally written using capital Phi (Φ):

$g = -\nabla\Phi$

The use of lowercase phi (φ) in the PPC formulation and capital Phi (Φ) in the General Relativistic weak-field formulation is purely a matter of notation. It does not, by itself, indicate a physical difference between the potentials in the common weak-field limit.

When the PPC weak-field equation is shown to have the same mathematical form and normalization as the corresponding weak-field equation obtained from Einstein's field equations, the two potentials can be identified in that limit:

$\varphi_{PPC} = \Phi_{GR}$

This identification is restricted to the weak-field correspondence. It does not mean that PPC and General Relativity are the same theory. They remain distinct theoretical frameworks, with potentially different fundamental equations, physical interpretations, and predictions beyond the common weak-field regime.

Therefore, the lowercase phi used in the PPC equation and the capital Phi used in the General Relativistic weak-field equations should be understood as different notational choices for the corresponding gravitational potential when the weak-field correspondence is being established.

1. $\nabla^2\Phi = 4\pi G\rho$

2. $g = -\nabla\Phi$

For the static, weak-field perfect-fluid limit, the two equations are:

1. $\nabla^2\Phi = 4\pi G(\rho + 3p/c^2)$

2. $g = -\nabla\Phi$

We can write this equation in small phi notation too.

For the static weak-field perfect-fluid case:
1. $\nabla^2\varphi = 4\pi G(\rho + 3p/c^2)$
2. $g = -\nabla\varphi$
Or
Weak field case:
1. $\nabla^2\varphi = 4\pi G\rho$
2. $g = -\nabla\varphi$